

Volume 03, Issue 6

Comparison of Panel Cointegration Tests

Deniz Dilan Karaman Örsal
Humboldt University Berlin

Abstract

The main aim of this paper is to compare the size and size-adjusted power properties of four residual-based and one maximum-likelihood-based panel cointegration tests with the help of Monte Carlo simulations. In this study the panel-rho, the group-rho, the parametric panel-t, the parametric group-t statistics of Pedroni(1999) and the standardized LR-bar statistic of Larsson et al.(2001) are considered. The simulation results indicate that the panel-t and the standardized LR-bar statistic have the best size and power properties among the five panel cointegration test statistics evaluated.

This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk". I thank Peter Pedroni, Helmut Herwartz and Ralf Brüggemann for helpful comments and suggestions.

Citation: Deniz Dilan Karaman Örsal, (2008) "Comparison of Panel Cointegration Tests", *Economics Bulletin*, Vol. 3 no.6 pp. 1-20.

Submitted: Jan 22 2008. **Published:** January 27, 2008.

1 Introduction

In this study two within-dimension-based (panel- ρ and parametric panel- t) and two between-dimension-based (group- ρ and parametric group- t) panel cointegration statistics of Pedroni (1999) are compared with the maximum-likelihood-based panel cointegration statistic of Larsson et al. (2001)¹. Among Pedroni's tests the group- ρ statistic is chosen, because Gutierrez (2003) demonstrates that this test statistic has the best power among the test statistics of Pedroni (1999), Larsson et al. (2001) and Kao (1999). The parametric group- t statistic is selected, because the data generating process used for the simulation study, is appropriate for parametric ADF-type tests. In addition to this, the within-dimension versions of these statistics (i.e. panel- ρ and parametric panel- t) are considered in order to be able to compare them with their between-dimension versions.

The main idea for the residual-based tests of Pedroni (1999) is to test for the existence of a unit root in the residuals of a spurious regression, where the existence of a unit root in the residuals will account for no cointegration between the components of the model. On the other hand, Larsson et al. (2001) presents a maximum-likelihood-based panel test for the cointegrating rank in the panel vector error-correction (VEC) model. They propose a standardized LR-bar test based on the mean of the individual rank trace statistic of Johansen (1995).

In this paper with a simulation study, the changes in size and size-adjusted power of the panel cointegration tests in finite samples is examined, when time and cross-section dimensions and various parameters in the data generating process vary, e.g. the correlation parameters between the disturbances to the stationary and non-stationary parts of the DGP for each cross-section.

The outline of the paper is as follows: Section 2 indicates how the DGP of Toda (1995) is modified for the panel data, and the section 3 gives a description of the simulation study. Section 4 is devoted to the interpretation of the simulation results. Conclusions are given in Section 5.

2 Data Generating Process

The Monte Carlo study is based on the data generating process (DGP) of Toda (1995), which has been used in several papers in the literature². The canonical form of the Toda process allows us to see the dependence of the test performance on some key parameters.

Let $y_{i,t}$ be a p -dimensional vector, where i is the index for the cross-section, t is the index for the time dimension and p denotes the number of variables in the model.

¹To save space the definition of the tests will not be given here. For the formulation of the test statistics we encourage the reader to look at the original papers. For an overview of the panel cointegration tests see, Banerje (1999), Baltagi and Kao (2000) etc..

²Lütkepohl and Saikkonen (2000), Saikkonen and Lütkepohl (1999, 2000), Hubrich et al. (2001), Trenkler (2006), Trenkler et al. (2006) etc.

The data generating process has the form of a VAR(1) process. The general form of the modified Toda process for a system of three variables in the absence of a linear trend in the data is,

$$y_{i,t} = \begin{pmatrix} \psi_a & 0 & 0 \\ 0 & \psi_b & 0 \\ 0 & 0 & \psi_c \end{pmatrix} y_{i,t-1} + \varepsilon_{i,t} \quad t = 1, \dots, T; \quad i = 1, \dots, N, \quad (1)$$

where the initial values of $y_{i,t}$, which can be represented as $y_{i,0}$ are zero. The error terms for each cross-section have the following structure:

$$\varepsilon_{i,t} = \begin{pmatrix} \varepsilon_{1i,t} \\ \varepsilon_{2i,t} \end{pmatrix} \sim i.i.d.N \left(0, \begin{pmatrix} I_r & \Theta \\ \Theta' & I_{p-r} \end{pmatrix} \right). \quad (2)$$

The true cointegrating rank of the process is denoted by r and $\varepsilon_{1i,t}$, $\varepsilon_{2i,t}$ are the disturbances to the stationary and non-stationary parts of the data generating process, respectively. Θ represents the vector of instantaneous correlations between the stationary and non-stationary components of the relevant cross-section.

Taking into account (1), when $\psi_a = \psi_b = \psi_c = 1$, a cointegrating rank of $r = 0$ is obtained. Thus, the data generating process becomes,

$$y_{i,t} = I_3 y_{i,t-1} + \varepsilon_{i,t}, \quad (3)$$

where $\varepsilon_{i,t} \sim i.i.d.N(0, I_3)$, which means that the process consists of three non-stationary components and these components are instantaneously uncorrelated. The VEC representation of (3) is:

$$\Delta y_{it} = \Pi_{i,t} y_{i,t-1} + \varepsilon_{i,t}. \quad (4)$$

Here, $\Pi_{i,t} = -(I_3 - A_{i1})$, and $A_{i1} = I_3$ represents the coefficient matrix of the VAR(1) process from (3). As $\Pi_{i,t}$ is a null matrix, (4) turns into:

$$\Delta y_{it} = \varepsilon_{i,t}.$$

With $|\psi_a| < 1$ and $\psi_b = \psi_c = 1$ the true cointegrating rank of the DGP is $r = 1$, and it is composed of one stationary and two non-stationary components, which can be formulated as,

$$y_{i,t} = \begin{pmatrix} \psi_a & 0 \\ 0 & I_2 \end{pmatrix} y_{i,t-1} + \varepsilon_{i,t} \quad \text{with} \quad \varepsilon_{i,t} \sim i.i.d.N \left(0, \begin{pmatrix} 1 & \Theta \\ \Theta' & I_2 \end{pmatrix} \right), \quad (5)$$

where $\Theta = (\theta_a, \theta_b)$ and $|\theta_a|, |\theta_b| < 1$.

The cointegrating rank of the process is $r = 2$, when ψ_a and ψ_b are less than unity in absolute value and $\psi_c = 1$. This can be represented in matrix form as,

$$y_{i,t} = \begin{pmatrix} \psi_a & 0 & 0 \\ 0 & \psi_b & 0 \\ 0 & 0 & 1 \end{pmatrix} y_{i,t-1} + \varepsilon_{i,t} \quad \text{with} \quad \varepsilon_{i,t} \sim i.i.d.N \left(0, \begin{pmatrix} I_2 & \Theta' \\ \Theta & 1 \end{pmatrix} \right), \quad (6)$$

where $\Theta = (\theta_a, \theta_b)$ and θ_a, θ_b are less than unity in absolute value. This process consists of one non-stationary and two stationary components and these components are correlated, when at least θ_a or θ_b is different from zero.

When $|\psi_a|, |\psi_b|$ and $|\psi_c| < 1$ the DGP is an $I(0)$ process, and the cointegrating rank is $r = 3$, which can be represented as,

$$y_{i,t} = \Psi y_{i,t-1} + \varepsilon_{i,t}, \quad (7)$$

where $\Psi = \text{diag}(\psi_a, \psi_b, \psi_c)$ and $\varepsilon_{i,t} \sim i.i.d.N(0, I_3)$.

3 Simulation Study

In order to see how the performance of the tests is affected by some key parameters, throughout the simulation study the time and cross-section dimensions, ψ_a, ψ_b, ψ_c parameters and the correlation parameters θ_a and θ_b will vary.

The correlation parameters θ_a and θ_b take the values $\{0, 0.4, 0.7\}$ and ψ parameters take the values $\{0.5, 0.8, 0.95, 1\}$. The value 0.95 for ψ parameters will help us to see how the tests react when the cointegrating rank of the process is near zero. The performance of the tests under the assumption of no instantaneous correlation between the disturbances is checked by $\theta_a = \theta_b = 0$.

To compare the results with Larsson et al. (2001), for the cross section dimension N I employ $N = \{1, 5, 10, 25, 50\}$ and for the time dimension T I employ $T = \{10, 25, 50, 100, 200\}$, using 1000 number of replications for each experiment. While generating the random error terms, seeded values are used and the first 100 observations are deleted, so that the starting values are not anymore zero. The tests were programmed in GAUSS 5.0.

For the tests of Pedroni (1999) the regression equation with a heterogeneous intercept is considered. To determine the lag truncation order of the ADF t -statistics, the step-down (sd) procedure and the Schwarz lag order selection criterion (sc) are used. The maximum lag order for the panel- t and group- t statistics is limited to 3, because this is the maximum lag order allowing an efficient estimation for small time dimensions, e.g $T = 10$. To select the lag order for the non-parametric panel- ρ and group- ρ statistics a kernel estimator is used. For the maximum-likelihood-based test statistic no VAR model lag order selection criterion is used, since the data is generated using a VAR(1) process. Only the null of no cointegration hypothesis is tested, as the residual-based tests cannot test for the number of cointegrating relations.

4 Interpretation of the Simulation Results

The importance of this simulation study lies in the DGP. In this study, the DGP is based on AR processes and covers the small sample properties of the residual-based tests, when there is more than one independent variable in the DGP, which is not done by Pedroni (1995)³. The most interesting results for empirical size and power properties of the panel cointegration tests are presented on figures 1 -7⁴.

4.1 Empirical Size Properties

On figures 1(a) and 1(b) group- ρ and panel- ρ statistics are not demonstrated, because their empirical sizes are always zero for $T = 10, 25$ and $N \geq 1$, which means that the true hypothesis of no cointegration can never be rejected. The severe size distortions for the other test statistics when T is small and N is large, can easily be recognized from figures 1(a) and 1(b), (e.g. the empirical sizes of the test statistics except for the panel- and group ρ statistics are unity, when $T = 10$ and $N \geq 25$). It is also obvious from the figures that when T is small, the test statistics become more oversized with increasing N . The reason for this may be the fact that the asymptotic first and second moments are used in order to standardize the statistics. Thus, the appropriate moments from the finite sample distribution of the test statistics should be used for the short time series⁵. This points out the fact that these tests are not appropriate, if the time dimension is much smaller than the cross-section dimension. However, on figures 1(d) and 1(e) it is clear that when T and N dimensions increase, the empirical sizes of the standardized LR-bar and panel- t test statistics approach to the nominal size level of 5%, especially for $T = 200$ and $N \geq 5$. On the other hand, the empirical sizes of the group- ρ and panel- ρ statistics are around 5% when $T = 100$, $N \geq 5$ and $T = 50$, $N \geq 5$, respectively. The size distortions of group- t , panel- t and standardized LR-bar test statistics decrease for fixed N , if T increases.

4.2 Size-Adjusted Power Properties

The relevant graphs for the size-adjusted power results are demonstrated starting with Figure 2. When $\psi_a, \psi_b, \psi_c \in \{0.5, 1\}$ and there is no correlation, just the results for $T = 10$ will be discussed, because the powers of all the test statistics approach unity for $T \geq 25$ and $N \geq 10$. If $T = 10$, standardized LR-bar and group- t statistics have the lowest power for the true cointegrating ranks of $r = 1$ and 2. In figures 2(a)-(b) the panel- ρ statistic has the highest power reaching 0.891 and 0.681 for $r = 1$ and 2, respectively. If the true cointegrating rank is $r = 3$ as on Figure 2(c), the power of rejecting the null-hypothesis

³Pedroni (1995) uses a bivariate model with MA(1) error process for his simulation study.

⁴In the figures, "sc" is the abbreviation for Schwarz lag selection Criterion, whereas "sd" denotes the step-down lag selection method.

⁵In addition to this, Hanck (2007, 2006) explains the increase of the size distortion with the increase in N as the cumulative effect of the small size distortions in the time series.

of no cointegration is the highest for the standardized LR-bar statistic (i.e. 0.53), whereas the group- t statistics have the lowest power (i.e. 0.04).

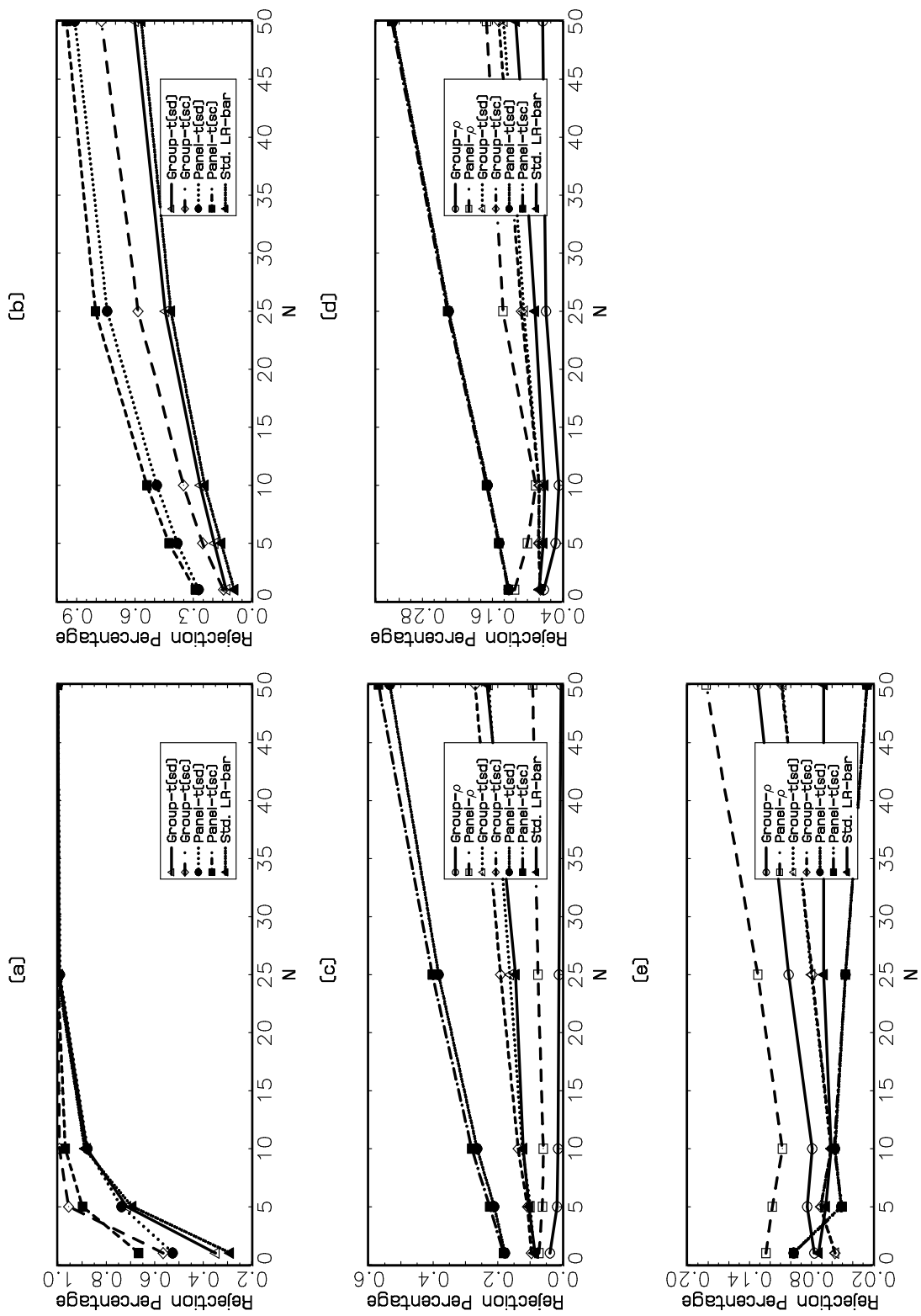


Figure 1: Small sample size of tests for (a) $T = 10$, (b) $T = 25$, (c) $T = 50$, (d) $T = 100$, and (e) $T = 200$.

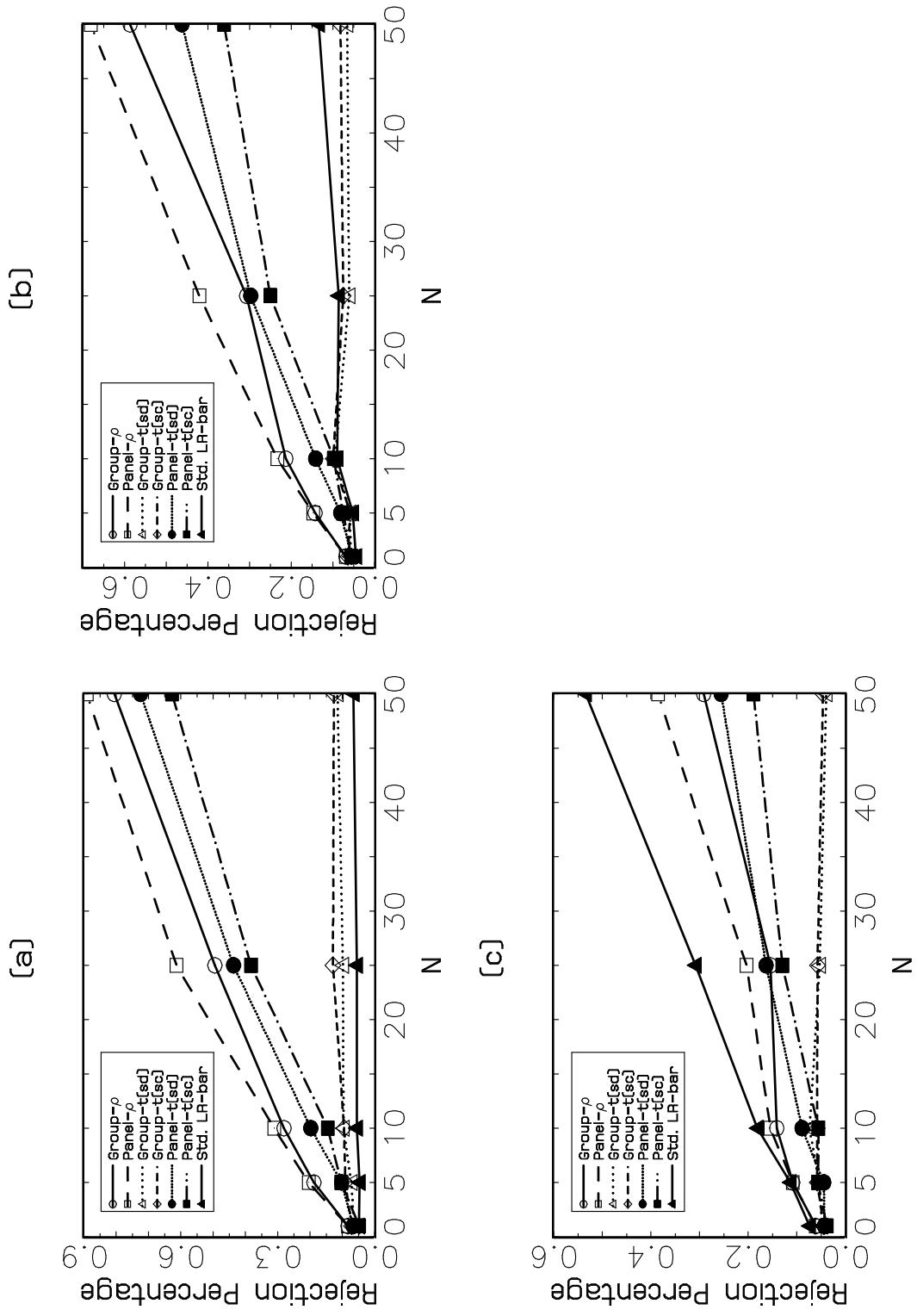


Figure 2: Small sample size-adjusted power of tests for $T = 10$, when (a) $\theta_a = \theta_b = 0$, $\psi_a = 0.5$, $\psi_b = \psi_c = 0$, (b) $\theta_a = \theta_b = 0$, $\psi_a = \psi_b = 0.5$, $\psi_c = 0$, (c) $\theta_a = \theta_b = 0$, $\psi_a = \psi_b = \psi_c = 0.5$.

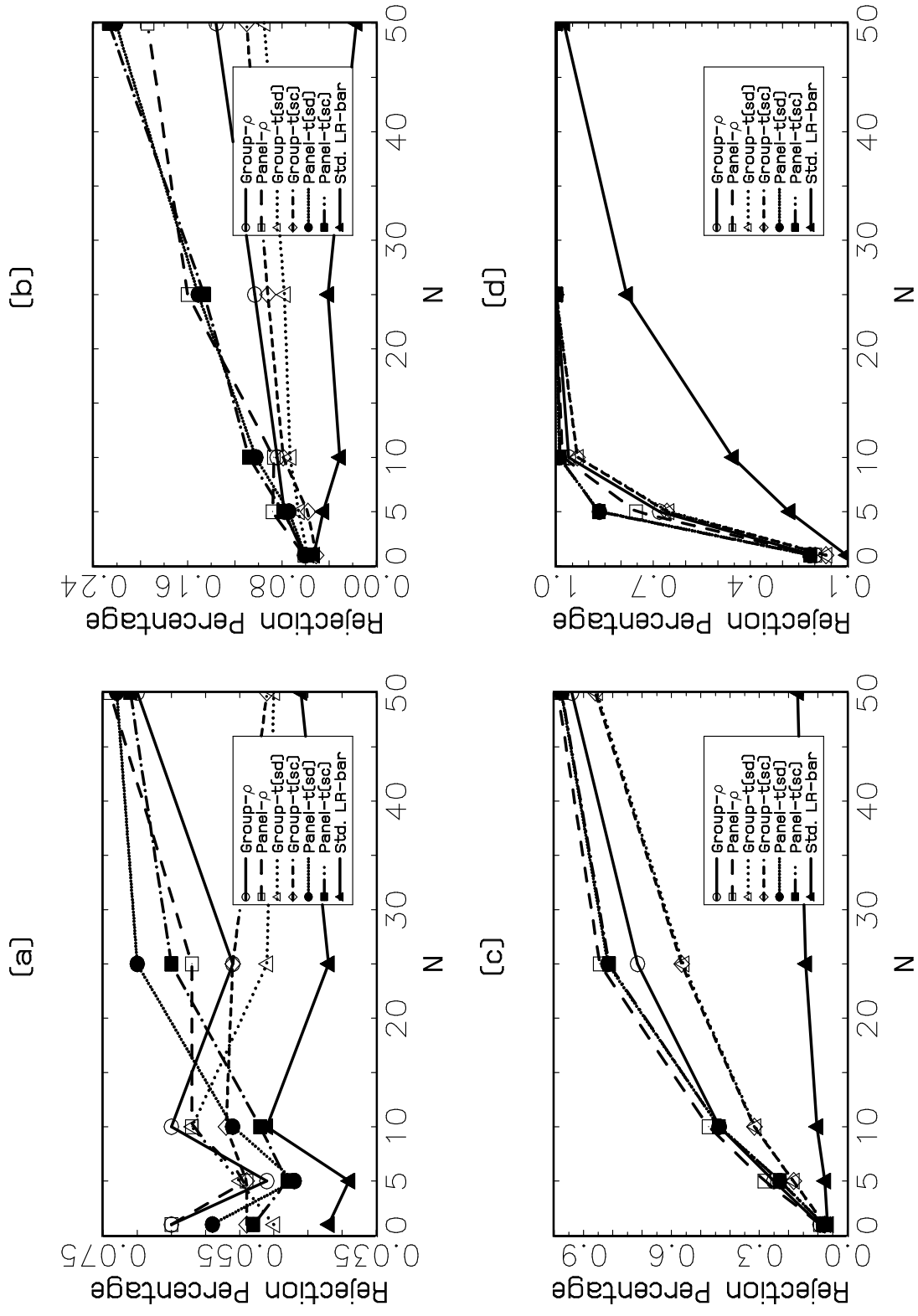


Figure 3: Small sample size-adjusted power of tests for $\theta_a = \theta_b = 0$, $\psi_a = 0.95$ and $\psi_b = \psi_c = 1$, when (a) $T = 10$, (b) $T = 25$, (c) $T = 100$ and (d) $T = 200$.

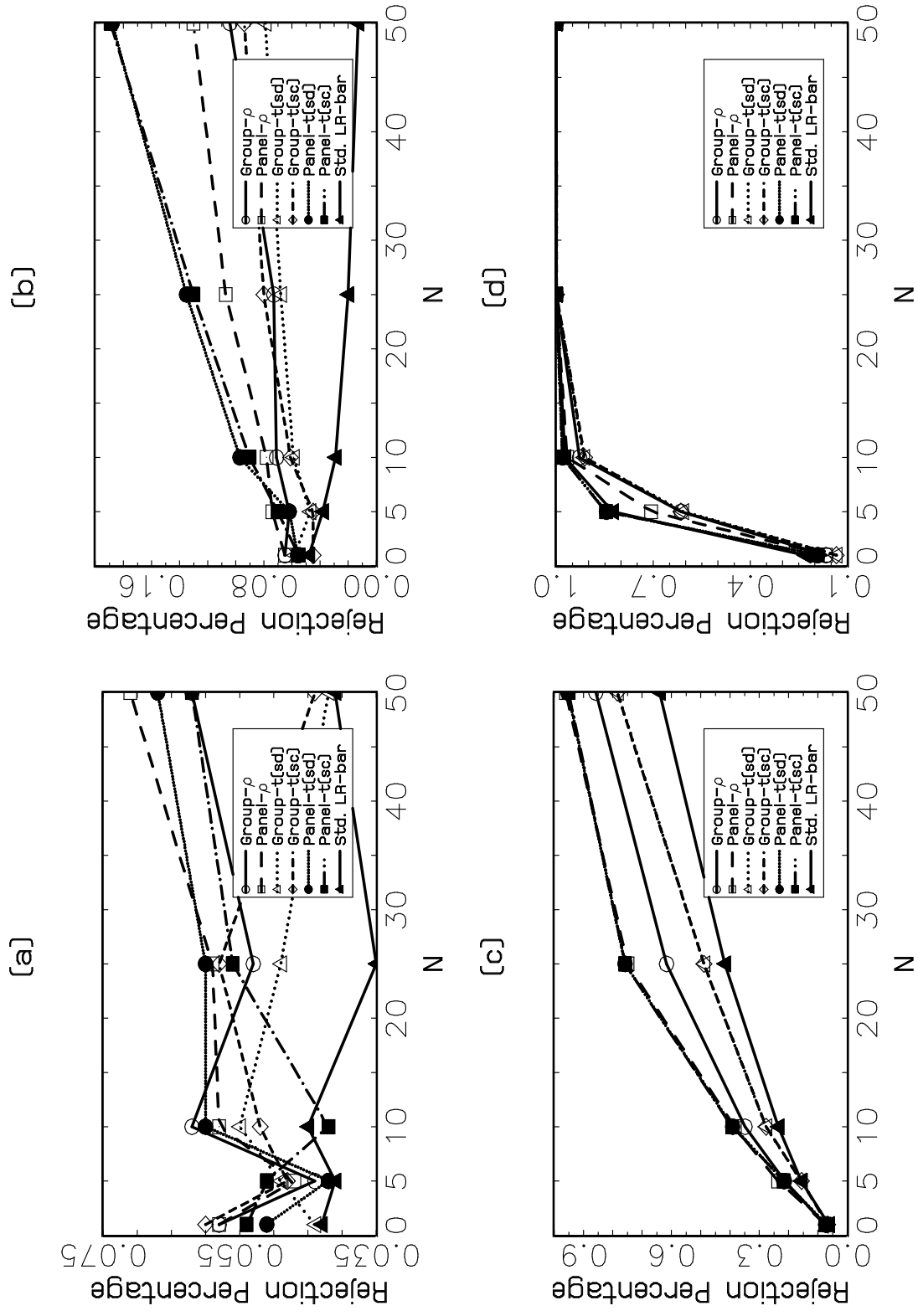


Figure 4: Small sample size-adjusted power of tests for $\theta_a = \theta_b = 0$, $\psi_a = \psi_b = 0.95$ and $\psi_c = 1$, when (a) $T = 10$, (b) $T = 25$, (c) $T = 100$ and (d) $T = 200$.

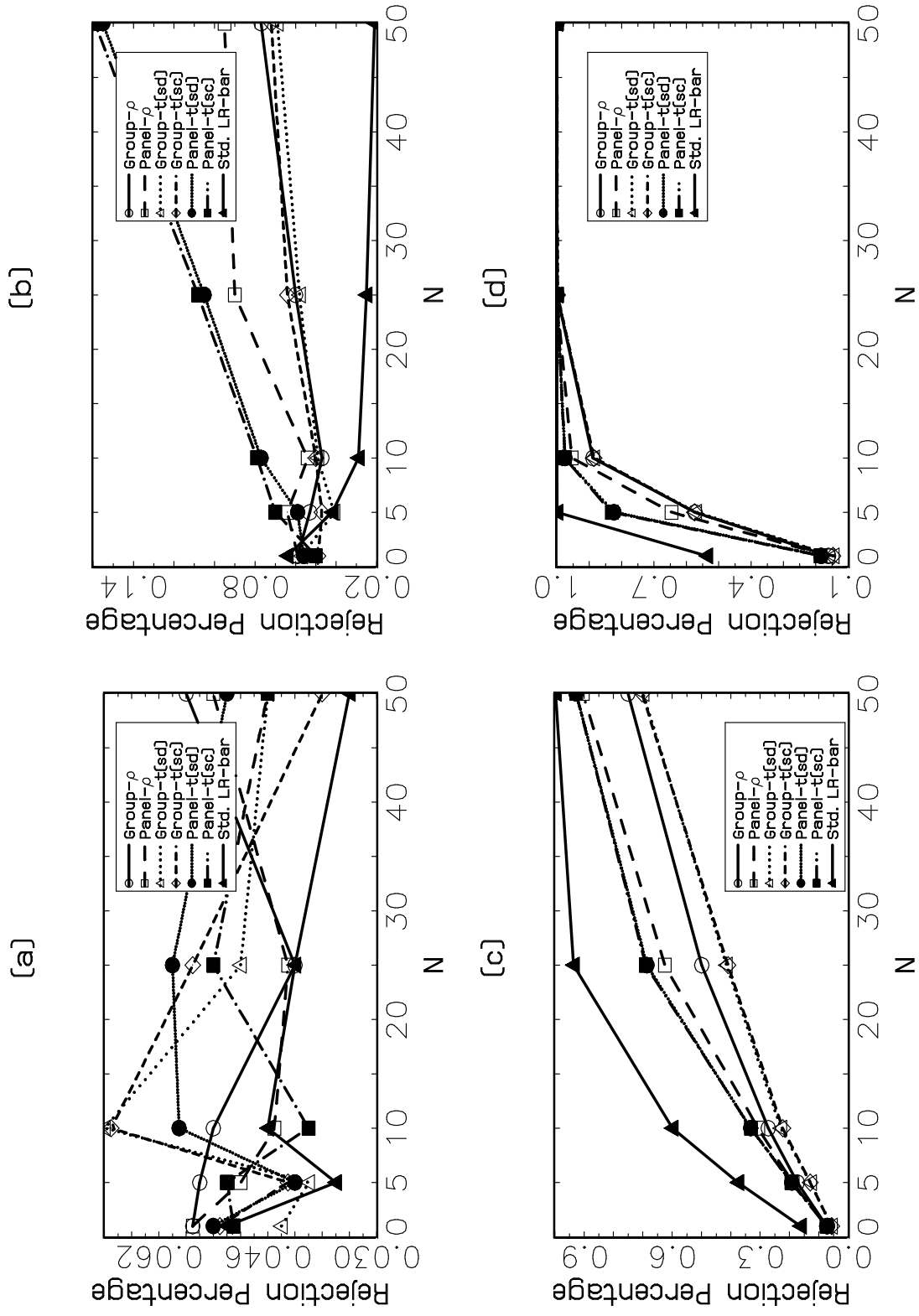


Figure 5: Small sample size-adjusted power of tests for $\theta_a = \theta_b = 0$, $\psi_a = \psi_b = \psi_c = 0.95$, when (a) $T = 10$, (b) $T = 25$, (c) $T = 100$ and (d) $T = 200$.

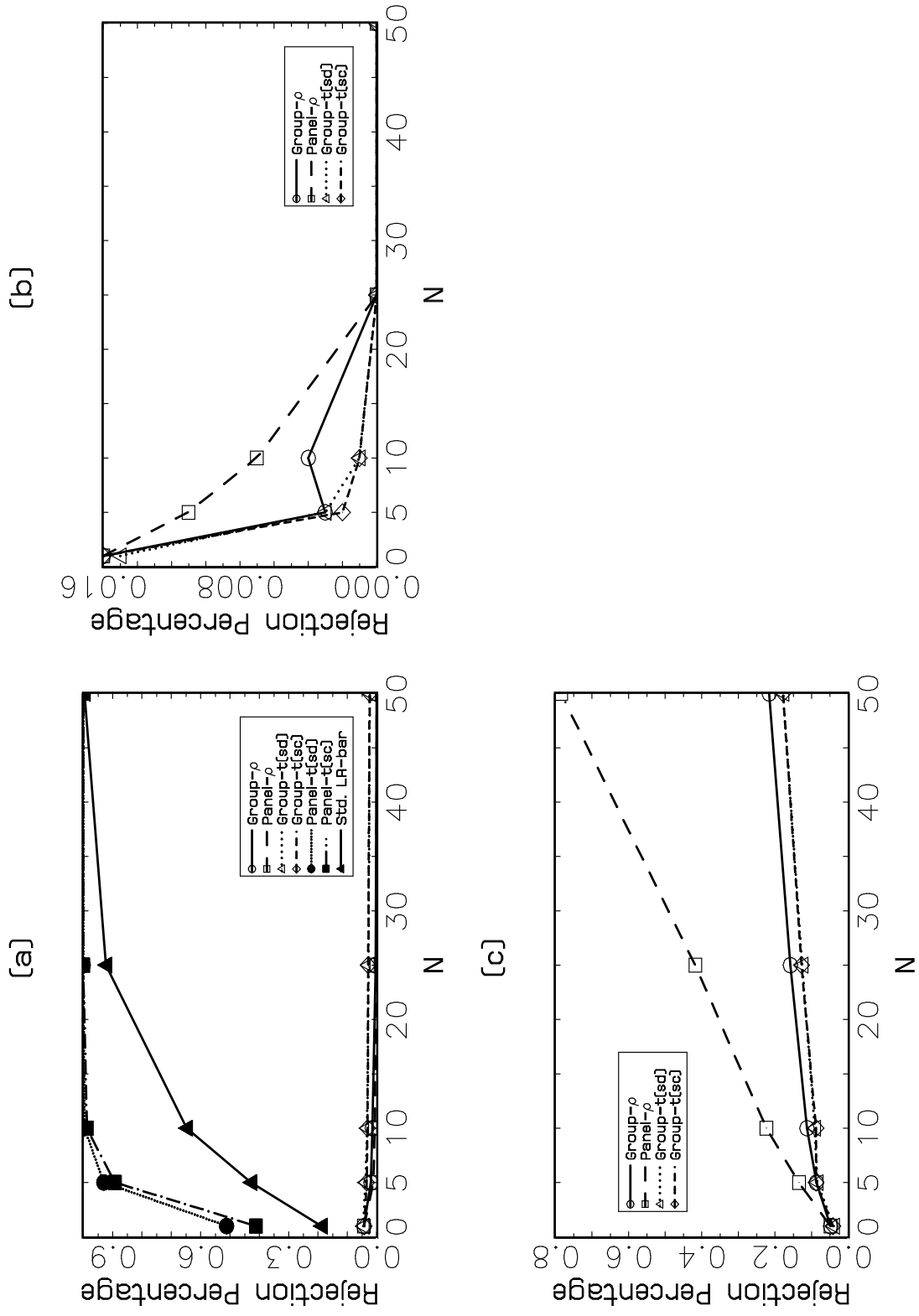


Figure 6: Small sample size-adjusted power of tests for $\theta_a = \theta_b = 0.7$, $\psi_a = 0.95$ and $\psi_b = \psi_c = 1$, when (a) $T = 10$, (b) $T = 100$ and (c) $T = 200$.

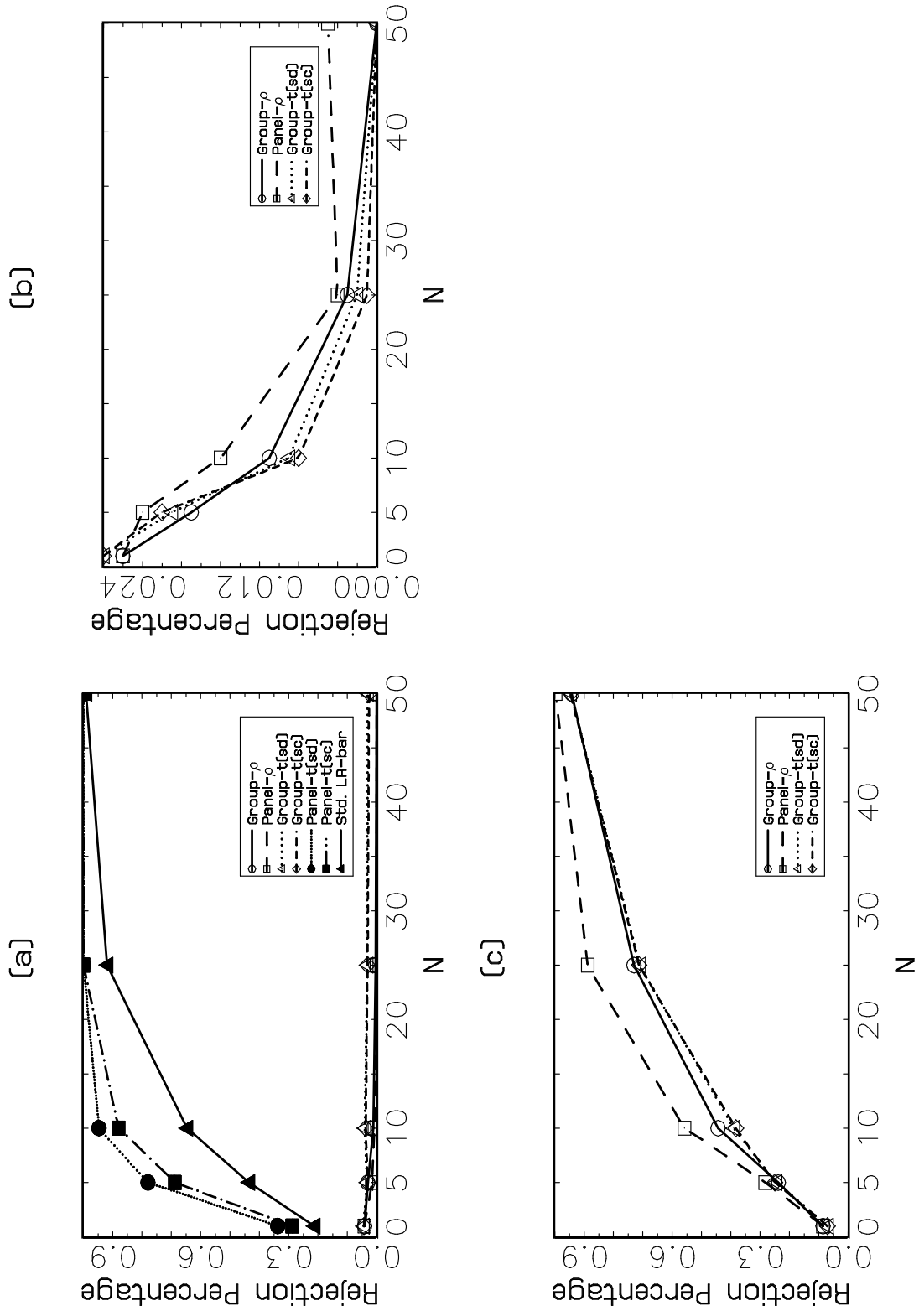


Figure 7: Small sample size-adjusted power of tests for $\theta_a = \theta_b = 0.7$, $\psi_a = \psi_b = 0.95$ and $\psi_c = 1$, when (a) $T = 10$, (b) $T = 100$ and (c) $T = 200$.

As there is not much difference in the size-adjusted power results when ψ parameters increase to 0.8, the results for this case are not presented here ⁶. If the ψ parameters are near unity with 0.95 and $T = 10$, the powers of all the test statistics are at most 0.074 for $r = 1, 2$ and 3, which can be observed on figures 3(a), 4(a) and 5(a). Figures 3(b) and (c) indicate that the standardized LR-bar test statistics have the lowest power and the panel- ρ and panel- t test statistics have the highest power. With the true cointegrating assumption of $r = 2$, figures 4(b) and (c) present that the maximum-likelihood-based test statistic has the lowest power again. The powers of all the test statistics converge to unity for high T and N dimensions, which confirms what the theory concludes. One interesting outcome of the Monte Carlo study belongs to the case when $T = 100$ and $r = 3$. For this case the standardized LR-bar test statistic has the highest power among all the test statistics. This eye-catching difference can be observed on Figure 5(c).

In order to understand how the test statistics behave under the assumption of correlated error terms, only the case with the highest correlation parameters is discussed, because the power results do not change drastically, if the correlation between the error terms is not high or ψ parameters are low. For $\psi_a, \psi_b, \psi_c \in \{0.95, 1\}$ the powers of the standardized LR-bar and the panel- t statistics approach unity even if $T = 10$. On the other hand, the powers of the other test statistics are near zero for small T dimensions (see figures 6(a) and 7(a)). For $T \in \{100, 200\}$ only the group- ρ , panel- ρ and group- t statistics are illustrated, because the other test statistics converge to unity faster. The power of rejecting the cointegrating rank of zero for group- ρ and group- t statistics cannot go to unity when the true rank is $r = 1$, even if T and N dimensions are high. The power of all the test statistics converge to unity, if the true cointegrating rank is $r = 2$ (see Figure 7(c)). Moreover, the powers of group- t and panel- t test statistics are not much different for Schwarz and step-down lag selection methods, when T and N increase.

5 Conclusion

With the extensive simulation study in Section 4, it can be concluded that the panel- t test statistic has the best size and size-adjusted power properties. On the other hand, the other three residual-based panel cointegration test statistics; group- ρ , panel- ρ and parametric group- t have poor size-adjusted power results, if the correlation and ψ parameters are high (e.g. when $\theta_a = \theta_b = 0.7$ and $\psi_a = \psi_b = 0.95$, respectively).

The second best test statistics which has the best size and power properties is the standardized LR-bar statistic. It has better size-adjusted power, if the correlation parameter is high and the ψ parameter is around unity. The empirical size of the standardized LR-bar statistic is around 5% like the panel- t test statistic, when both T and N increase, especially if T increases faster than the cross-section dimension just as the theory points out. Moreover, it has high power, when T is large. It should also be emphasized that the size and size-adjusted power results of the residual-based panel cointegration tests can depend on the choice of the dependent variable. Here, the first variable of the DGP has

⁶The results can be supplied by the author on request.

been taken as the dependent variable for the residual-based panel cointegration tests. For a future study the procedure in Larsson et al. (2001) can be extended to a new maximum-likelihood-based panel cointegration test statistic with a constant and a linear trend in the data, because this is currently not possible in the context of Larsson et al. (2001) test.

References

- Baltagi, B. and Kao, C. (2000), “Nonstationary panels, cointegration in panels and dynamic panels: A survey”, Working Paper 16, Center for Policy Research.
- Banerje, A. (1999), “Panel data unit roots and cointegration: An overview”, *Oxford Bulletin of Economics and Statistics*, **61**, 607–629.
- Gutierrez, L. (2003), “On the power of panel cointegration tests: A Monte Carlo comparison”, *Economics Letters*, **80**, 105–111.
- Hanck, C. (2006), “The error-in-rejection probability of meta-analytic panel tests”, Universität Dortmund, SFB 475 Technical Report 46/06, Ruhr Graduate School in Economics.
- Hanck, C. (2007), “A meta analytic approach to testing for panel cointegration”, Universität Dortmund, SFB 475 Technical Report 02/07, Ruhr Graduate School in Economics.
- Hubrich, K., Lütkepohl, H., and Saikkonen, P. (2001), “A review of systems of cointegration tests”, *Econometric Reviews*, **20**, 247–318.
- Johansen, S. (1995), *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford University Press: Oxford.
- Kao, C. (1999), “Spurious regression and residual-based tests for cointegration in panel data”, *Journal of Econometrics*, **90**, 1–44.
- Larsson, R., Lyhagen, J., and Løthgren, M. (2001), “Likelihood-based cointegration tests in heterogeneous panels”, *Econometrics Journal*, **4**, 109–142.
- Lütkepohl, H. and Saikkonen, P. (2000), “Testing for the cointegrating rank of a VAR process with a time trend”, *Journal of Econometrics*, **95**, 177–198.
- Pedroni, P. (1995), “Panel cointegration; asymptotic and finite sample properties of pooled time series tests, with an application to the PPP hypothesis”, Indiana University, Working Paper Series in Economics 95-013, Ruhr Graduate School in Economics.
- Pedroni, P. (1999), “Critical values for cointegration tests in heterogeneous panels with multiple regressors”, *Oxford Bulletin of Economics and Statistics*, **61**, 653–670.
- Saikkonen, P. and Lütkepohl, H. (1999), “Local power of likelihood ratio tests for the cointegrating rank of a VAR process”, *Econometric Theory*, **15**, 50–78.

- Saikkonen, P. and Lütkepohl, H. (2000), “Testing for the cointegrating rank of a VAR process with structural shifts”, *Journal of Business and Economic Statistics*, **18**, 451–464.
- Toda, H. Y. (1995), “Finite sample performance of likelihood tests for cointegrating ranks in vector autoregressions”, *Econometric Theory*, **11**, 1015–1032.
- Trenkler, C. (2006), “Bootstrapping systems cointegration tests with a prior adjustment for deterministic terms”, Humboldt Universität zu Berlin, SFB 649 Discussion Paper Series 2006-12, School of Economics and Business.
- Trenkler, C., Saikkonen, P., and Lütkepohl, H. (2006), “Testing for the cointegrating rank of a VAR process with level shift and trend break”, Humboldt Universität zu Berlin, SFB 649 Discussion Paper Series 2006-67, School of Economics and Business.